# Assignment 8

Coverage: 16.2, 16.3 (most) in Text.

Exercises: 16.2 no 11, 17, 20, 24, 29, 32, 37, 41, 42, 43. 16.3 no 2, 5, 9, 11, 15, 16, 18, 20, 27, 29.

Hand in 16.2 no 32, 42; 16.3 no 11, 16, 20 by Nov 8.

### Supplementary Problems

1. Let $\mathbf{c}$ be a parametric curve from $[a, b]$ to $C$. Another parametric curve $\gamma$ is called a reparametrization of $\mathbf{c}$ if $\gamma(t) = \mathbf{c}(\varphi(t))$ where $\varphi$ is a continuously differentiable map from $[\alpha, \beta]$ one-to-one onto $[a, b]$. Show that
$$\int_a^b f(\mathbf{c}(t))|\mathbf{c}'(t)|\,dt = \int_\alpha^\beta f(\gamma(t))|\gamma'(t)|\,dt\ .$$

2. Let $F = (F_1, \cdots, F_n)$ be a smooth vector field in an open region in $\mathbb{R}^n$. Show that if it is conservative, then the necessary conditions hold
$$\frac{\partial F_i}{\partial x_j} = \frac{\partial F_j}{\partial x_i}\ , \quad \forall i, j.$$

3. Let $\mathbf{F}$ be a smooth vector field in the entire space $\mathbb{R}^n$. Show that
$$\Phi(x, y, z) = \int_0^1 \mathbf{F}(tx, ty, tz) \cdot (x\mathbf{i} + y\mathbf{j} + z\mathbf{k})\,dt\ ,$$
defines a potential function for $\mathbf{F}$ provided it passes the component test.

4. Let $C$ be the oriented curve runs from the origin to $(2, 0)$ along the cardioid $r = 1 + \cos\theta$ in the upper half plane. Find the work done of $\mathbf{F} = (\sin xy + xy\cos xy)\mathbf{i} + x^2\cos xy\mathbf{j}$ along $C$.